### Example: Truth tables

Let A and B be statements. Every statement can be either true or false. In the truth table,A B there is one line for each possible combination of truth values for the operands that occur in the considered formula. For a propositional formula containing the operands and ,

AAAA is false and BBBB is false. the following combinations are possible:

is true and is false.

is false and is true.

is true and is true.

the operands A and B contains four lines, with each line corresponding to a combination. No other combinations exist. Thus a truth table for a propositional logical formula with The first two columns of the truth table therefore look like this:

A 0 corresponds to the truth value false as described above, a 1 stands for true. The other columns of the truth table depend on the concrete propositional formula to be considered.

that displays the value of the expression A ∧ B, depending on the selected combination For example, let us look at the truth table for the conjunction. Here there is a third column of truth values for this line.

The first line shows the truth value A B A ∧ B AA ∧ B has if both B A and B are false. The second lineA B

shows the truth value of if is true and is false. The third line contains the value in case is false and is true. The fourth and last line shows the case that both and are true.

As a further example the truth table for the implication follows:

This will be illustrated by a simple example with three operands A ∨ (B ∧ C) A, B, and C. Truth tables are more interesting when more complex formulas are examined. In these cases, further columns are added in which individual partial expressions are evaluated. We consider the formula . Because the formula links three statements, we consider three operands ∧ Cof are added, because there are now more combinations for possible truth values — instead22 = 4 possible combinations there are now A, B, and C, each of which can take two truth values, true or23 = 8 possible combinations (because weB must add another column to the truth table for the third operand. Besides this, more lines

false, each). In addition, we add another column in which we first evaluate the bracket (

) before we determine the truth value of the entire expression. By breaking down com-

example, the truth table for A ∨ (B ∧ C) could look like this: plex formulas in this way into small “bites” and evaluating them piece by piece (from left to right in the truth table), the determination of truth values is simplified considerably. For

We have seen that, depending on the assignment of the variables, a propositional logical expression can usually be true or false. However, there are also expressions that are always true regardless of the assignment of the variables. Such propositional formulas are called tautologies.

Tautology

A statement or formula that is true in every possible sense is known as a tautology.